AES 2320 Pt 1 Re-Exam 30 June 2017

1. A Bingham plastic flows if $\tau_{rz}|_{r=R} \geq \tau_0$ (BSL1 Ex. 2.3-2). In other words, [see note on next page]

$$\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R \geq \tau_0 \qquad (I)$$

What is $\Delta\mathcal{P} = (\mathcal{P}_0 - \mathcal{P}_L)$?

$p_0 = 10^5\ Pa + \rho g (0.1)$ (entrance 10 cm below top of mayo)

p_L = unknown

$\Delta\mathcal{P} = p_0 - p_L - \rho g(0.3)$ – flow goes up, against gravity, 0.3 m

$= 10^5 - p_L - \rho g (0.2)$ [Note entrance (p_0) is at bottom of tube]

(from Eq I) $10^5 - p_L - \rho g(0.2) \geq \tau_0 \frac{2L}{R}$

$p_L \leq 10^5 - \rho g (0.2) - \tau_0 \frac{2L}{R}$

$\leq 10^5 - (910)(9.82)(0.2) - 100\left(\frac{2(0.3)}{0.002}\right)$

$\leq 10^5 - 1787 - 3\cdot 10^4$

$\leq 6.82 \cdot 10^4$ (almost 1/3 bar vacuum)

(Note that μ_0 is irrelevant to the onset of flow)

2. a) We don't know v, so we don't know Re. Trial + error:

$f = \frac{4}{3} g D \frac{1}{v_\infty^2} \frac{\rho_s - \rho}{\rho}$ (BSLK Eq. 6.1-7)

$= \frac{4}{3}(9.82)\, 10^{-4} \frac{1}{v^2}\left(\frac{2500-1000}{1000}\right) = \frac{0.001964}{v^2}$

$Re = \frac{Dv\rho}{\mu} = \frac{10^{-4} v\ 1000}{0.001} = 100 v$

I would guess with a small particle, Re is small.

Guess $v = 0.01$ m/s ; $Re = 1$; $f \cong 30 \rightarrow v = 0.0081$

$Re = 0.8$ $\quad \sim 35 \quad v = 0.0075$ m/s

0.75 can't read difference. Done

b) Now $f_V = \frac{0.000982}{v^2}$; $Re = 50 v$

Guess $v = 0.002$ m/s ; $Re = 0.2$ $\quad f \cong 130 \quad v = 0.0027$

0.27 $\qquad$ 0.0031

Slower velocity for smaller grains → longer time to settle. In a turbidite smaller grains are layered on larger grains.

3. There are two differences from BSL1 Sect. 2.2:

- Different BC at $x=\delta$; instead, $v_z = V \neq 0$ at $x=\delta$
- viscosity not constant.

Which change affects derivation first?

The integration of Eq. 2.2-12 to obtain Eq. 2.2-13 assumes μ is constant. This comes before the B.C. at $x=\delta$ (Eq. 2.2-15).

Therefore, the last eq. that can be used directly is Eq. 2.2-13

Depending on whether you interpret symbol μ as necessarily a constant, one might say eq. 2.2-11 is the last eq. that applies. I accepted any answer between 2.2-11 and 2.2-13. By eq. 2.2-14, though, the integration has been done assuming μ is a constant.

Note on solution to problem 1:

Because we are interested in the onset of flow, you don't need the equation for v_z (BSL1 2.3-25+26) or Q (BSL1 2.3-10). Moreover, note that eq. 2.3-25 applies only for $r \geq r_0$. At the onset of flow, $r_0 = R$, and $\tau_R = \tau_0$. Either way, one is driven back to the condition that $\tau_{rz}|_{r=R} = \tau_0$.

4. a) A sudden constriction and kinetic energy are involved.

Use macro mech E balance (BSL I Eq. 7.5-12)

Surface "1" just upstream of tube; "2" at outlet.

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}v^2$

$g(h_2 - h_1) = 0.2\,(9.8\text{?})$

$\frac{p_2 - p_1}{\rho} = \frac{10^5 - 2\cdot10^5}{1000} = -100$

no work in or out

fittings: one sudden constriction.

$\frac{1}{2}v^2(0.45)$

friction along tube: $2v^2 \frac{0.2}{0.005} f$ (III)

Putting it together:

$\frac{1}{2}v^2 + 1.96 - 100 = -2v^2 40 f(Re) - 0.225v^2$

$v^2(\frac{1}{2} + 80f + 0.225) = 98.04$ (II)

Need to solve by trial + error. $Re = \frac{Dv\rho}{\mu} = \frac{0.005\,v\,1000}{0.001} = 5000v$

Guess $v = 1$ m/s; $\frac{k}{D} = \frac{0.02}{5} = 0.004$, $Re = 5000$ $f \doteq 0.01$ (Fig 6.2-2)

$v^2(0.725 + 80(0.01)) = 98.04 \rightarrow v = 8.0$ m/s

$Re = 4.0\cdot10^4$ $f \approx (0.0082) \rightarrow$ $v = 8.4$ m/s

$Re = 4.2\cdot10^4$ ≈ 0.0082 again. Can't tell difference. Done

b) if D increases, the only term in Eq. II to change is the friction-factor term, Eq. (III). It gets smaller,* which makes the bracketed term in Eq. II smaller. That means v increases. He should use the largest tube dia. practical, as long as his tank can continue to supply water at constant pressure.

* in 2 ways: first, at fixed roughness factor $\frac{k}{D}$, f decreases a bit as Re increases. More important, $\frac{k}{D}\downarrow$ as $D\uparrow$, so $f\downarrow$.

Why does this seem to contradict experience? If the ability to supply water at fixed p were limited (in other words, fixed Q, not fixed p_1), then a narrower tube would be better.